Correlation and Prediction of Gas-Liquid Holdups in Inclined Upflows

A correlation is developed for predicting in-situ (or true) liquid holdups for two-phase, gas-liquid slug flows in inclined pipes. The correlation is based on experimental data collected in pipes ranging from 2.54 to 7.94 cm in diameter with the air-water system used.

The experimental data used to develop the correlation were collected at zero input liquid qualities, as previously discussed by Greskovich (1973), and extended over the entire range. These data revealed little diameter effect on holdup for diameters greater than 2.54 cm but a marked effect of angle of inclination. Predicted holdup values show good agreement with experimental data.

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SCOPE

The prediction of liquid holdup for two-phase, gas-liquid slug flows in pipelines is a must for design engineers. Correlations, for example, based on the early work of Lockhart and Martinelli (1949) for horizontal flows have been extended to inclined flows, and predictions by Flannigan (1958) for the contribution of hills to overall line pressure drops have markedly added to the technology, but both leave a lot to be desired.

The objective of the research reported here was to develop a holdup correlation for inclined two-phase flows using only operating parameters known a priori to the design engineer. The correlation was to be tested with data collected in pipes up to a diameter of 7.94 cm with the air-water system used. Rather than collect holdup data over the entire range of input liquid qualities, experimental data at zero input liquid qualities were obtained and extrapolated by using a new technique (Greskovich, 1973) based on the correlation of Guzhov et al. (1967). Once verified, the model and resulting correlation can be used for scale-up purposes.

CONCLUSIONS AND SIGNIFICANCE

It has been shown that in-situ, or true, liquid holdups for inclined two-phase slug flows can be easily obtained over the range by experimentally measuring such holdups at zero input liquid qualities and then extrapolating the data based on Greskovich (1973). Experimental holdup data from a 7.94 cm diameter pipe were compared with data from smaller diameter pipes, and, based on the correlation of true liquid holdup against input liquid quality, by using mixture Froude number and angle of inclination as parameters, there appeared to be little direct effect of pipe diameter for pipes greater than 2.54 cm other than that taken into account by the mixture Froude number.

A model for predicting liquid holdup in inclined flows was developed, based on Bonnecaze et al. (1971), and agreed very well with the experimental data and extrapolations presented. The correlation can be easily used since only a knowledge of operating parameters is necessary to obtain predicted values of in-situ liquid holdup.

It is necessary to accurately predict in situ (or true) liquid holdup during two-phase, gas-liquid flows in long distance pipelines for design purposes. Since a major portion of the overall line pressure drop is due to ascending portions of the pipelines, the accurate prediction of inclined true liquid holdups is extremely important for design and subsequent line operation. In view of the total number of miles of such pipes already built, and those in the planning stages, the prediction of inclined holdups becomes, in most probability, the single most important task for the design engineer.

At present, there are few articles in the literature which consider flows; however, their apparent importance is gaining more and more attention. Early work by Flannigan (1958) demonstrated the contribution of the hills to the overall line pressure drop. Subsequent works by a number of investigators, as summarized by Gouse (1963, 1964, 1966) have discussed the importance of uphill flows but rarely presented any design data or correlations useful for scale-up. A number of works have appeared in the Russian literature concentrating in the area of two-phase, gas-liquid pipeline analysis. Articles have appeared by Mamaev (1965), Gallyamov and Goldzberg (1967, 1968), Isupov and Mamaev (1968), and Guzhov et al. (1967), representing a sizable research effort in this area. Similarly, a number of American works, such as those by Bonnecaze et al. (1969, 1971), Greskovich (1972), Bonderson (1969), and many contributions by Griffith et al. (1971, 1973), have also considered the subject of inclined two-phase flows.

A particular method for correlating actual liquid holdup with flowing liquid quality has been discussed by most of the above Russian investigators and expanded upon by Greskovich et al. (1969), Greskovich and Shrier (1971), and Greskovich (1972). The correlation utilizes only mixture Froude number as a parameter and has been shown by Greskovich and Shrier (1971) to be quite accurate for horizontal flows. The only reported use of such a correlation for inclined flows is made by Guzhov et al. (1967) for a pipe inclined 9 deg. from the horizontal.

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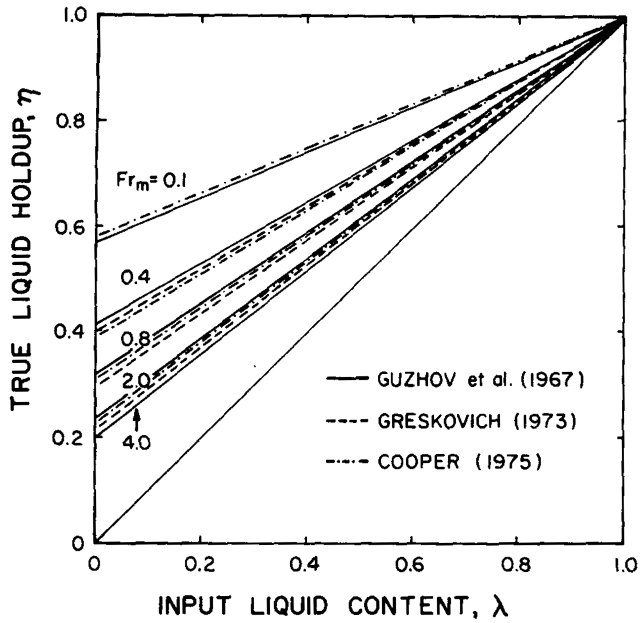

Fig. 1. Dependence of true liquid content on input liquid quality and mixture Froude number at 9 deg.

More recently, Greskovich (1973) developed a new technique for obtaining true liquid holdups for inclined (positive inclinations) two-phase flows. The technique consisted of measuring liquid holdups at zero liquid flow rates, corresponding to bubbling gas upward through a quiescent liquid. By recognizing that straight lines correlated the holdup data based on Guzhov et al. (1967), once the intercept was determined experimentally it was easy to extend the lines to predict true liquid holdups over the entire range of input liquid qualities. This technique was further discussed by Gregory (1974), and it was found that the liquid holdup predictions with the Guzhov et al. (1967) correlation used compared well with other data for in situ liquid fractions greater than 0.25. For values less than 0.25, the Lockhart-Martinelli (1949) correlation was recommended. These recommendations were made in view of the lack of additional data, especially for larger diameter pipes and other systems.

DATA FROM LARGER DIAMETER PIPES

Since it became apparent that Greskovich (1973) was recommending a new procedure for obtaining in situ liquid holdups and suggesting that the correlation was strongly dependent on angle of inclination, it was necessary to further test these assumptions in our laboratories at larger pipe diameters. Liquid holdups were obtained in a 7.94 cm diameter pipe, approximately 939 cm long, which was inclined from the horizontal in a range from 1 to 10 deg. as described by Cooper (1975). It is noted in Figure 1 that prior data from Guzhov et al. (1967) with a pipe approximately 5.08 cm in diameter and Greskovich (1973) with a pipe approximately 2.54 cm in diameter are plotted at an inclination of 9 deg. from the horizontal. Data from the 7.94 cm pipe are also plotted on Figure 1 and were obtained exactly as those described by Greskovich (1973). That is, the only experimental datum point is that located at the intercept ($\lambda = 0$), and the lines represent extrapolations from the experimental values of η at $\lambda = 0$ to the point $\eta = 1$, $\lambda = 1$. Data for mixture Froude numbers between 0.1 and 4 are considered, since this represents an optimum operating range for commercial pipelines, and within this range, slug flow exists.

It is surprising to note that within experimental error there does not appear to be any additional diameter effect on the true liquid holdups predicted by using the correlation in Figure 1. It could be assumed that the mixture Froude number takes this into account; however, when data from pipe diameters smaller than 2.54 cm are plotted on Figure 1, there is a sizable deviation from the lines presented there. In fact, the data used by Gregory (1974) included liquid holdup values obtained from pipe diameters less than 2.54 cm, and, based on these values used at low true liquid holdups, a sizable deviation from those predicted by Figure 1 resulted.

It is proposed in the work of Cooper (1975) and here that there is indeed a diameter effect on the prediction of true liquid holdup by using the correlation presented in Figure 1 up to diameters of approximately 2.54 cm. Beyond this diameter, the effect diminishes to zero. The data presented by Greskovich (1973), Cooper (1975), and the data here seem to warrant this hypothesis. Furthermore, it has been proposed by Griffith (1974) that as slugs of gas move up through inclined pipes, the diameter of the bubble nose, not the pipe diameter, is the correlating parameter. When the pipes are small, the pipe diameter is approximately equal to the bubble diameter; however, when the pipe diameter is greater than approximately 2.54 cm, the slug of gas does not completely bridge the pipe diameter. As the pipe diameter increases, the bubble diameter at the leading side does not correspondingly increase. Additional evidence supporting this proposal has been obtained by Cooper (1975).

In a 7.94 cm diameter pipe, true liquid holdups were measured at angles of inclination of 1, 2, 6, 9, and 10 deg. from the horizontal. These data were compared to data from Greskovich (1973) for a pipe diameter of 2.54 cm, and the result was very similar to that found on Figure 1. Within experimental error of approximately 0.02 in the value of the true liquid holdup, data from both studies were approximately coincident for each angle of inclination; however, there was a noticeable increase in liquid holdups as the angles increased.

A CORRELATION FOR INCLINED FLOWS

A model for predicting true liquid holdups for two-phase inclined flows has been previously described by Bonnecaze et al. (1971), recognizing that for most operating pipelines the flow regime is slug flow. Based on contiunity, it was shown for uphill slug flow that

$$\eta = 1 - (1 - \lambda)/(C_1 + U_{BR}/U_{NS}) \qquad (1)$$

Equation (1) relates the true liquid holdup η to the flowing input liquid quality λ, the bubble rise velocity of the gas bubble U_{BR}, and the no-slip or mixture velocity U_{NS}. The bubble rise velocity is defined for vertical flows by Taylor (1950):

$$U_{BR} = C_2 \sqrt{gD} \qquad (2)$$

As discussed previously, the diameter in Equation (2) is the bubble nose diameter as defined by Taylor and has been assumed equal to the pipe diameter (Griffith and Wallis, 1961). By combining Equation (2) with the mixture Froude number, and by assuming that for inclined flows the bubble rise velocity is a direct function of the angle of inclination $f(\theta)$ (Runge and Wallis, 1965), it is shown that

$$U_{BR} = C_2 \, (U_{NS}/Fr_m^{1/2}) \, [f(\theta)] \qquad (3)$$

Combining Equations (1) and (3) we get

$$\eta = 1 - \frac{(1 - \lambda)}{C_1 + \dfrac{C_2 f(\theta)}{Fr_m^{1/2}}} \qquad (4)$$

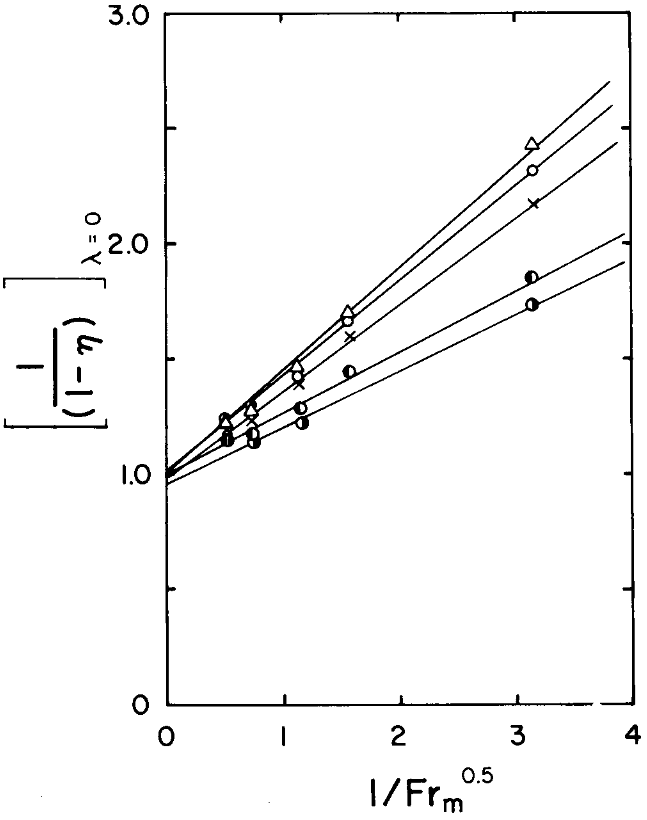

Fig. 2. Cross-correlation plot at various angles of inclination and $\lambda = 0$.
◑— 1 deg.
◐— 2 deg.
×— 6 deg.
○— 9 deg.
△—10 deg.

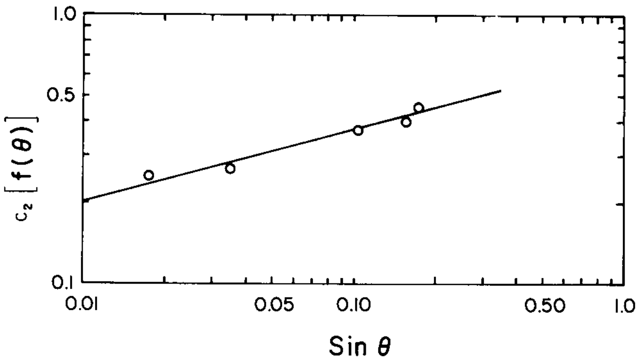

Fig. 3. Correlation of theta function with angle of inclination.

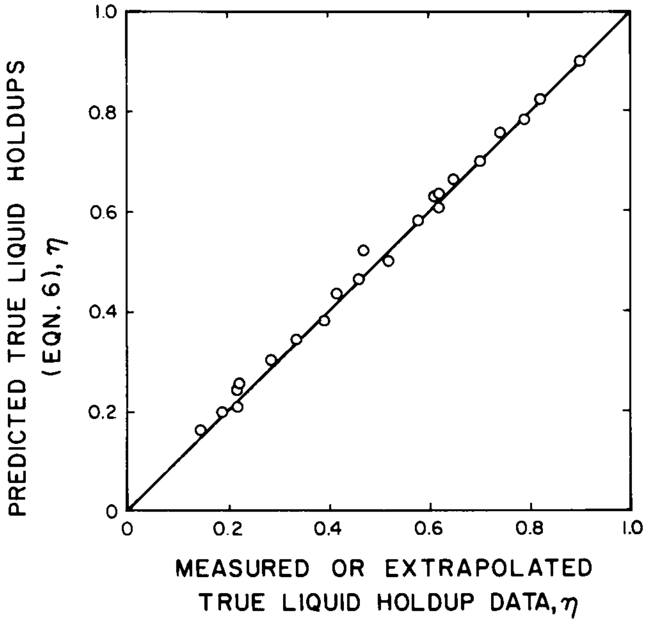

Fig. 4. Comparison of predicted and measured true liquid holdups.

Since Equation (4) is dimensionless, a dimensionless function $f(\theta)$ was sought which would correlate both the data of Greskovich (1973) and Cooper (1975). It was assumed for this work that this function should take the form $(\sin\theta)^b$. The constant C_1, as defined by many investigators, such as Bonnecaze et al. (1971), has been shown to be in the range 1.15 to 1.35. The constant C_2 loses its identity in Equation (4), since $C_2 f(\theta)$ was obtained by solving for the general function $a(\sin\theta)^b$, where the constant a does not necessarily equal the constant C_2 in the Taylor bubble rise equation [Equation (2)].

Experimental data from Greskovich (1973) for a 2.54 cm diameter pipe and Cooper (1975) for a 7.94 cm diameter pipe at $\lambda = 0$, corresponding to flowing gas through a quiescent liquid, at various mixture Froude numbers (for this case the flowing mixture reduces to gas only) and various angles of inclination were used to test Equation (4). By rearranging Equation (4) at $\lambda = 0$, it can be shown that

$$[1/(1-\eta)]_{\lambda=0} = C_1 + \frac{C_2 f(\theta)}{Fr_m^{1/2}} \tag{5}$$

From Figure 2, it can be noted that a plot of $[1/(1-\eta)]_{\lambda=0}$ vs. $1/Fr_m^{1/2}$ at constant angles of inclination yielded varying slopes but intercepts between approximately 0.96 and 1.05. On this basis, an average value of C_1 was chosen to be 1.0. Similarly, Figure 3 was used to determine the coefficients for the function $a(\sin\theta)^b$ by plotting the slopes of Figure 2 [equal to $C_2 f(\theta)$ for each θ as defined by Equation (5)] against $\sin\theta$ on log-log paper. From the slope of Figure 3, a value for b was estimated to be 0.263, and from the intercept the value for a was 0.671.

PROPOSED CORRELATION TESTED OVER THE ENTIRE RANGE OF λ'S

The correlation resulting from the derivation and experimental data collected at zero input liquid qualities, $\lambda = 0$, was evaluated over the entire range of λ values based on the data and extrapolations of Guzhov et al. (1967), Greskovich (1973), and Cooper (1975). The equation in its final form is

$$\eta = 1 - (1-\lambda)/[1.0 + 0.671\ (\sin\theta^{0.263}/Fr_m^{0.5})] \tag{6}$$

When we use mixture Froude numbers of 0.1, 0.4, 0.8, 2.0, and 4.0, angles of inclination of 1, 2, 6, 9, and 10 deg. from the horizontal, and values of input liquid content of 0.1, 0.3, 0.5, and 0.7, the predictions using Equations (6) are compared with data on Figure 4. The agreement between these values is well within $\pm 5\%$ for most of the data, but for low values of η the deviations could be as high as $\pm 10\%$.

CONCLUSIONS

A relatively large amount of data were used in this study representing three different pipe diameters and five different angles of inclination for the air-water system. True liquid holdups were collected at zero input liquid qualities experimentally and uniquely extrapolated over the range of input liquid qualities as previously described. By comparing this technique with limited available literature data, that of Guzhov et al. (1967) at 9 deg., it has been shown that the technique is reliable. Furthermore, when comparing true liquid holdup data from two different pipe diameters, we can determine that the mixture Froude numbers adequately account for variations in pipe diameter, and any additional diameter effect is minimal when using the data correlation of η vs. λ at various angles of inclinations using mixture Froude number as a parameter for pipe diameters greater than 2.54 cm.

Finally, based on this extrapolative technique and data from Guzhov et al. (1967), a correlation has been developed which proved to be most reliable for predicting true liquid content for inclined slug flows. Since the correlation has been tested with pipe diameters up to 7.94 cm, it is recommended for general use with those traditional reservations regarding the character of the gas-liquid system. That is, all the data used in this work and those data used to test the proposed correlation were based on the air-water system, and for general use, this correlation should be tested with other gas-liquid systems.

NOTATION

A = pipe cross-sectional area, cm^2
C_1, C_2 = constants in Equations (1) and (2), respectively
D = pipe diameter, cm
$f(\theta)$ = some function of angle of inclination
Fr_m = mixture Froude number, U_{NS}^2/Dg
g = gravitational constant
Q_G, Q_F = gas and liquid flow rates, respectively, cm^3/s
U_{NS} = no-slip or mixture velocity, $Q_G + Q_F/A$, cm/s
U_{BR} = bubble rise velocity given by Equation (2), cm/s

Greek Letters

α = gas void fraction
η = true liquid holdup, fraction
$\eta_{\lambda=0}$ = true liquid holdup at zero liquid rate
θ = angle of inclination from the horizontal
λ = input liquid quality, $Q_F/Q_F + Q_G$

LITERATURE CITED

Bonderson, L. S., "An Analysis of the Pressure Drop and Void For a Two-Phase Slug Flow in Inclined Pipes," M.S. thesis, Mass. Inst. Technol., Cambridge (1969).

Bonnecaze, R. H., W. Erskine, and E. J. Greskovich, "Holdup and Pressure Drop For Two-Phase Slug Flow in Inclined Pipelines," paper presented 62nd Annual Meeting of Am. Inst. Chem. Engrs., Washington, D. C. (1969).

———, "Holdup and Pressure Drop FOR Two-Phase Slug Flow in Inclined Pipelines," *AIChE J.*, **17**, 1109 (1971).

Cooper, W. T., "Prediction of Liquid Holdup For Two-Phase, Gas-Liquid Slug Flow in an Inclined Pipeline," M.S. thesis, Bucknell Univ., Lewisburg, Pa. (1975).

Flannigan, O., "Two-Phase Gathering Systems," *Oil Gas J.*, **132**, (Mar. 10, 1958).

Gallyamov, A. K., and V. L. Goldzberg, "Normal Flow Depth During Motion of a Gas-Liquid in Descending Line Sections," *Neft i Gaz*, **1**, 70 (1967).

———, "Flow of Gas-Liquid Mixtures in the Descending Sections of a Pipeline," *ibid*, **11**, 67 (1968).

Gouse, S. W., Jr., "An Index to the Two-Phase, Gas-Liquid Flow Literature," Mass. Inst. Technol. Reports *DSR 8734-1* (1963); *DSR 8734-4* (1964); and *DSR 8734-6* (1966).

Gregory, G. A., "Comments on the Prediction of Liquid Holdup For Gas-Liquid Flow in Inclined Pipes," *Can. J. Chem. Eng.*, **52**, 463 (1974).

Greskovich, E. J., "Slug Frequency in Horizontal Gas-Liquid Slug Flow," *Ind. Eng. Chem. Process Design Develop.*, **11**, 318 (1972).

———, "Holdup Predictions For Stratified Downflow of Gas-Liquid Mixtures," *ibid.*, 81.

———, and A. L. Shrier, "Pressure Drop and Holdup in Horizontal Slug Flow," *AIChE J.*, **17**, 1214 (1971).

———, and R. H. Bonnecaze, "True Gas Content For Horizontal Gas-Liquid Flow," *Ind. Eng. Chem. Fundamentals*, **8**, 591 (1969).

Greskovich, E. J., "Prediction of Gas-Liquid Holdup For Inclined Flows," *AIChE J.*, **19**, 1060 (1973).

Griffith, P., and G. Singh, "Determination of the Pressure Drop and Optimum Pipe Size For a Two-Phase Slug Flow in an Inclined Pipe," *ASME Paper No. 70-Pet.-15* (1971).

Griffith, P., P. C. Hon, C. W. Lau, and J. F. Pearson, "Two-Phase Pressure Drop in Inclined and Vertical Pipes," Report No. *DSR 80063-81*, Heat Transfer Laboratory, Mass. Inst. Technol., Cambridge (1973).

Griffith, P., and G. B. Wallis, "Two-Phase Slug Flow," *Trans. ASME J. Heat Transfer*, **83**, 307 (1961).

Griffith, P., Private Communication (1974).

Guzhov, A. I., A. A. Mamayev, and G. E. Odishariya, "A Study of Transportation in Gas-Liquid Systems," *10th Intern. Gas Conf.*, Hamburg, Germany (1967).

Isupov, G. P., and A. A. Mamayev, "Velocity Field of a Split Gas-Liquid Turbulent Flow," *Neft i Gaz*, **10**, (1968).

Lockhart, R. W., and R. C. Martinelli, "Proposed Correlation of Data For Isothermal Two-Phase, Two-Component Flow in Pipes," *Chem. Eng. Prog.*, **45**, 39 (1949).

Mamaev, V. A., "Some Problems in the Hydrodynamics of Joint Transport of Gas and Liquid," *Intern. Chem. Eng.*, **5**, 318 (1965).

Runge, D. E., and G. B. Wallis, "The Rise Velocity of Cylindrical Bubbles in Inclined Tubes," Tech. Report No. *NYO-31114-8*, Dartmouth College, Hanover, N. H. (1965).

Taylor, G., and R. M. Davis, "The Mechanics of Large Bubbles Rising Through Extended Liquids and Through Liquids in Tubes," *Proc. Royal. Soc.*, **A-200**, 375 (1950).

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An Unsteady State Method for Measuring the Permeability of Small Tubular Membranes

An experimental method is described for measuring the permeability of small tubular (hollow fiber) membranes by means of an unsteady state diffusion experiment. Permeability data obtained by this method are reported for experimental collagen and commercial cellulose acetate membranes for urea, sucrose, and polyethylene glycol (MW ~ 4 000).

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